



Looking for multiple equilibria when geography matters: German city growth and the WWII shock

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Abstract

Based on the methodology of Davis and Weinstein, we look for multiple equilibria in German city growth. By taking the bombing of Germany during WWII as an example of a large, temporary shock, we analyze whether German city growth is characterized by multiple equilibria. In doing so, we allow for spatial interdependencies between cities. The main findings are twofold. First, multiple equilibria are present in German city growth. Our evidence supports a model with two stable equilibria. Second, the inclusion of geography matters. The evidence for multiple equilibria is weaker when spatial interdependencies are not taken into account.

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1. Introduction

Notwithstanding their prominence in modern economic theory, it turns out to be rather difficult to assess the empirical relevance of models with multiple equilibria. This is also true for modern location theory of which the new economic geography (NEG) is a prime example. Ever since Krugman [14], the notion of multiple equilibria has been a key feature of the NEG liter-

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ature. Given the prevalence of multiple equilibria, not only in NEG models, but also in other location theories, it seems remarkable that there is so little empirical research into the existence of multiple equilibria. A main reason for this lack of empirical evidence is that tests for multiple equilibria call for the existence of large, temporary shocks and these shocks are rare.

Using the bombing of Japanese cities during WWII as their case of a large, temporary shock, Davis and Weinstein [6] are among the first to test for the presence of multiple equilibria. Their study is a sequel to Davis and Weinstein [5], in which they analyse for the case of Japan whether the WWII bombing affected the relative growth of cities in the post-WWII period. They find that this is not the case. This conclusion also holds, to some degree, for the case of (western) German cities and the WWII shock, as is shown by Brakman, Garretsen, and Schramm [4]. Based on these studies the conclusion seems to be that even in the face of huge shocks such as WWII, city growth is not characterized by multiple equilibria. After some time, city growth returns to its “pre-shock” level, thereby substantiating the stability of the initial equilibrium (Head and Mayer [12, p. 2662]).

Based on the findings in Davis and Weinstein [5] and Brakman, Garretsen, and Schramm [4], such a conclusion about the empirical irrelevance of multiple equilibria may, however, be premature (see also Gabaix and Ioannides [9]). First, it is important to note that NEG models typically only predict a switch to a new equilibrium for certain ranges of the key parameters. So the evidence of persistence of the initial, pre-shock equilibrium can still be consistent with a model of multiple equilibria. Second, geography or location is not an issue in Davis and Weinstein [5]. As a test of NEG models this abstraction is problematic since in NEG models spatial linkages between locations are crucial. Third, and very importantly for our present purposes, before dismissing the possibility of multiple equilibria we should test more explicitly for this possibility. Davis and Weinstein [5] test for a single, unique equilibrium, because the model specification does not allow for multiple equilibria. This last issue is taken up by Davis and Weinstein [6]. They develop an innovative method that examines necessary conditions for multiple equilibria for the case of Japanese city growth and the WWII shock. A rejection of these necessary conditions would be a more powerful rejection of the presence of multiple equilibria than the implicit rejection in Davis and Weinstein [5].¹

The methodology in Davis and Weinstein [6] can, however, also be criticised for the fact that it does not include geography or location. In their analysis there are no spatial interdependencies between cities. In this paper we not only apply the methodology developed by Davis and Weinstein [6] to the strategic bombing of German cities during WWII, but also explicitly include geography in our empirical analysis. It turns out that the inclusion of geography is critical to our finding that German city growth is characterized by multiple equilibria. The main contributions of the paper are thus twofold. We test for multiple equilibria in German city growth following the WWII shock and we do so by explicitly allowing geography to matter. Our evidence supports a model with two stable equilibria.

The paper is organized as follows. In the next section we will present the theoretical background for the model that will be used in the estimations. Section 3 briefly describes the extent of the destruction of German cities during WWII. It demonstrates that this constitutes a large, exogenous and temporary shock. Section 4 describes the empirical methodology and derives our

¹ An alternative approach to test for multiple equilibria is developed by Redding et al. [16]. They analyze the location of airport hubs in Germany and find evidence that after WWII Berlin was not able to hold on to its position as main airport hub, Frankfurt became the main airport hub. After German re-unification in 1990, Frankfurt retained its position and the authors find evidence that supports the idea of multiple equilibria in airport location in Germany.

main empirical specifications. The data and our main findings are discussed in Section 5. It is shown that geography is very important for our results. Section 6 concludes and provides some suggestions for further research.

2. Theoretical background

Davis and Weinstein [6] *loosely* base their analysis on the NEG literature and in particular on the first NEG model by Krugman [14]. The analysis of agglomerating and spreading forces that determines the stability of equilibria in Krugman [14] and subsequent NEG models is well known and we do not dwell on it here.² As is also emphasized by Davis and Weinstein [6], the two-region model of Krugman [14] has only three potentially stable equilibria (perfect spreading and full agglomeration in either region). In the real world, partial agglomeration seems to be the rule. Applied to cities this means that the underlying model should allow for multiple stable equilibria where multiple cities (agglomerations) exist that differ in size. The Krugman [14] model is therefore not well suited for our purposes since it typically only allows for either a mono-city (full agglomeration) or equally sized cities (spreading).³ It is, however, quite easy to extend the Krugman [14] model to include partial agglomeration as a stable equilibrium outcome. The basic framework as developed by Davis and Weinstein [6], summarized by Fig. 1, should therefore not be viewed as a direct test of Krugman [14] but rather as a general test of models, like some NEG models, that allow for multiple stable equilibria.⁴ In Appendix A we illustrate how NEG models can be linked to Fig. 1.

Given the assumption that the economy is initially in a stable equilibrium, a relocation or shock to the footloose labour force and firms can in theory have two implications for the equilibrium in NEG-type models. Either the shock is small and workers and firms return to their original location (that is to say to the initial stable equilibrium) or the shock is large enough to exceed the threshold of the nearest unstable equilibrium and the economy moves to another stable equilibrium. Following Davis and Weinstein [6] we can represent this adjustment process in a two-period growth space as depicted in Fig. 1. In order to facilitate comparison we adhere to their use of symbols. Our main variable of interest is the (log-)share of a city i in the total population, denoted by S_i (where we can suppress the subscript i since all cities are assumed to be alike in terms of their reaction to a particular city-specific shock of a given size; this is clearly an important (and restrictive) assumption as it means that the model underlying Fig. 1 is based on the notion of a representative city).⁵

² For a more extensive discussion of the Krugman [14] model see Fujita, Krugman and Venables [7] or Brakman, Garretsen and van Marrewijk [3]. Introducing additional (spreading) forces into this model increases the number of long-run equilibria and in particular opens the way to asymmetric, stable equilibria, that is stable equilibria with partial agglomeration.

³ Partial agglomeration can be an equilibrium but if it is then it is an *unstable* equilibrium in Krugman [14].

⁴ To give just one example, in NEG terms, Fig. 1 depicts three stable equilibria (Δ_1 , Δ_2 and Ω) and two unstable equilibria (the thresholds b_1 and b_2). By, for instance, adding an additional spreading force (congestion) to an otherwise unchanged Krugman [14] model one can end indeed up with three stable interior equilibria (interior meaning not full agglomeration) and two unstable interior equilibria (see Brakman, Garretsen, and van Marrewijk [3, Fig. 7.2C, p. 195]), see Appendix A for an illustration of this example.

⁵ Note that the through the use of log-shares, one reformulates the model in growth rates. This facilitates the empirical analysis. Furthermore, we assume that the situation depicted in Fig. 1 holds for each region or city: a shock of size X that changes the equilibrium for city A does the same for region or city B . Another important innovation of Davis and Weinstein [6], as compared to Davis and Weinstein [5], is that they do not only model city growth in terms of city

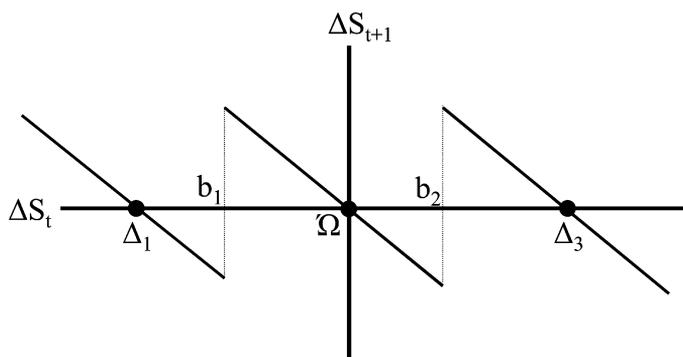


Fig. 1. Two-period growth representation of a model with three stable equilibria.

Let Ω indicate the initial stable equilibrium and let Δ_1 and Δ_3 depict two surrounding stable equilibria. If a shock that hits a city is too small to pass the thresholds b_1 or b_2 the city share will return to the initial equilibrium, implying that the second period change will completely undo the shock. This is indicated by the line through the origin.⁶ The period t is the period that includes the shock and the period $t + 1$ is the period in which the effects of the shock are undone, hence the slope of -1 (Davis and Weinstein [6, p. 7]). For an equilibrium to be stable it must be the case that in this two-period growth set-up there exists a period $t + 1$ that is long enough such that $\Delta S_t = -\Delta S_{t+1}$. It is an empirical matter if the time series for $t + 1$ are long enough such that $\Delta S_t = -\Delta S_{t+1}$. So, in practice even if the initial equilibrium is stable the slope could be less than 1 (in absolute terms). Of course, when the initial equilibrium is not stable it must be the case that $\Delta S_t \neq -\Delta S_{t+1}$ whatever the length of $t + 1$. We will return to this issue in Section 5.

If the shock is large enough to move the city's population share outside the range $b_1 - b_2$, a new stable equilibrium will be established and the economy will move to Δ_1 or Δ_3 depending on whether the city is hit by a negative or positive shock. But for each new stable equilibrium it must also be true in our two-period setting that $\Delta S_t = -\Delta S_{t+1}$. For the example of three stable equilibria shown in Fig. 1 these two possibilities are indicated by the two solid lines through Δ_1 and Δ_3 , both with a slope of minus unity. To sum up, in case of multiple equilibria we end up in the two period growth setting of Fig. 1 with a sequence of negative sloped lines with slope minus unity, where each line corresponds to a different stable equilibrium. Figure 1 can also be used to illustrate the difference between Davis and Weinstein [5], based on a model with a single equilibrium, and Davis and Weinstein [6], based on a model with multiple equilibria. To cite Davis and Weinstein [6]: "... the 2 period growth space thus provides a very simple contrast between a model of unique equilibrium versus one of multiple equilibria. In the case of a unique equilibrium, an observation should simply lie on a line with slope minus unity through the origin. In the case of multiple equilibria, we get a sequence of lines, all with slopes minus unity, but with different intercepts. Because in this latter case these lines have slope minus unity, the intercepts are ordered and correspond to the displacement in log-share space from the initial to the new equilibrium. These elements will be central when we turn to empirical analysis" (Davis and Weinstein [6, p. 8]).

population but also in terms of economic activity, the change of manufacturing activity across Japanese cities. We return to this in Section 6.

⁶ Note, that the origin has no special meaning other than indicating the initial situation (see also Eq. (10)).

Apart from the fact that we look at German cities instead of Japanese cities, the main difference compared to the Davis and Weinstein [6] set-up is that we do take spatial interdependencies into account. What happens to a particular city following the shock could in principle also be determined by the shock that hits other cities. In other words, in our empirical strategy to test the model underlying Fig. 1 we assume that $\Delta S_{t+1,i} = f(\Delta S_{t,i}, \Delta S_{t,j \neq i})$, where besides the time subscript, t , we also include a region or city subscript, i, j . For the empirical analysis this implies that we should not only look at the change in the size of a region or city in isolation, but also at the change of that region or city relative to the change in size of surrounding regions or cities corrected for inter-city distance. In Section 4 we will deal with these issues and our “geography” extension in more detail. But first we will briefly illustrate that the strategic bombing of German cities during WWII indeed provides a large, temporary shock, making it a suitable event to examine for the presence of multiple equilibria in German city growth.

3. The strategic bombing of Germany⁷

During the Second World War (WWII), allied forces heavily bombed Germany. At first these bombings had little effect on the German war economy, and the initially limited success of the air raids led to a change in bombing tactics. From March 1942 onwards, RAF bomber command followed a new bombing strategy. The emphasis now was on area-bombing, in which the centers of towns became the main target. The idea behind this strategy was that the destruction of cities would have an enormous and destructive effect on the morale of the people. The dislocation of workers would in any case disrupt industrial production, even if the factories themselves were not hit. Selected cities were not necessarily chosen because they were particularly important for the war economy, but because they were visible from the air. The economic importance of cities was often not decisive in the selection of targets, but the potential for destruction was.

On average 40% of the dwellings in the larger cities were destroyed (which is roughly comparable with the findings of Davis and Weinstein [5] for Japan). An estimated 410,000 people lost their lives due to air raids, and at least seven million people lost their homes. As a result in 1946 the population of many German cities was (in absolute terms) considerably lower than in 1939.

By the end of WWII many cities in Germany were significantly damaged. The destruction was primarily caused by the bombing campaign but in contrast to Japan, also by the invasion of Germany by the allied forces. These ground battles caused additional and severe damage to the cities that were in the path of the allied forces. Furthermore, by the end of the war the encounter between the Russian forces and the retreating German army led to an enormous inflow of refugees (*Vertriebene*) from former German territories and East European countries. Rough estimates indicate that between 11 and 14 million refugees had to find a new home in Germany (both in eastern and western Germany). This inflow of German refugees more than compensated for the loss of lives in Germany itself, but they could not return to the destroyed cities and had to find a place to stay in less damaged parts of Germany. The combined effect of the bombing campaign and the flow of refugees affected the city size distribution in Germany enormously (Brakman, Garretsen, and Schramm [4]). Based on this information our conclusion is that the strategic bombing of Germany during WWII and the consequences of the occupation of Germany by allied forces is an example of a large shock that could provide evidence for multiple equilibria in the sense of Fig. 1.

⁷ To a large extent the information in this section is based on Brakman, Garretsen, and Schramm [4] to which we refer for more details.

4. Model specification

As a formal test of the WWII shock on German city growth, we employ the methodology of Davis and Weinstein [5,6]. In Davis and Weinstein [5] the main hypothesis is whether the evidence supports the idea of a unique, stable equilibrium following the WWII shock. In terms of Fig. 1 they test whether it is true that the observations lie on a line with slope minus unity through the origin, that is the equilibrium depicted by Ω in Fig. 1. The possibility of other equilibria (in cases Δ_1 and Δ_3) is *not* taken into account in the analysis, which means that for the case of a unique equilibrium Fig. 1 reduces to one line through the origin with slope minus unity. In Davis and Weinstein [6] the model specification allows for the possibility of multiple equilibria as Fig. 1 illustrates.

For the case of a unique equilibrium, the approach is basically to test whether or not the growth of city size (with city size as a share of total population) follows a random walk. The relative city size s for each city i at time t can be represented by (in logs):

$$s_{i,t} = \Omega_i + \varepsilon_{i,t} \quad (1)$$

where Ω_i is the initial equilibrium size of city i and $\varepsilon_{i,t}$ represents city-specific shocks.

The persistence of a shock can be modeled as:

$$\varepsilon_{i,t+1} = \rho\varepsilon_{i,t} + v_{i,t+1} \quad (2)$$

where $v_{i,t}$ is independently and identically distributed (i.i.d.) and it is assumed that $0 \leq \rho \leq 1$.

By first differencing (1) and making use of (2) we get

$$s_{i,t+1} - s_{i,t} = (\rho - 1)v_{i,t} + (v_{i,t+1} + \rho(\rho - 1)\varepsilon_{i,t-1}). \quad (3)$$

If $\rho = 1$, all shocks are permanent and city growth follows a random walk; if $\rho \in [0, 1)$, the shock will dissipate over time. From Fig. 1 we know that for the initial equilibrium to be stable there must a period $t + 1$ that is long enough to ensure that $\rho = 0$. With $\rho = 0$ the shock has no persistence at all. For $0 < \rho < 1$ there is some degree of persistence. Of course, in the multiple equilibria version (see below), Fig. 1 also allows for another interpretation. Observe, after estimating Eq. (3), that $0 < \rho < 1$ could be consistent with multiple equilibria, in the sense that we could draw more than one line through the scatter plot, each with a slope of -1 . We should test for this alternative possibility, instead of only allowing for a unique equilibrium! For the simpler case of a unique equilibrium, however, we only have to test whether or not in terms of Fig. 1 the observations lie on a line through the origin with slope minus unity.

As Davis and Weinstein [5, p. 1280] note the value for the central parameter ρ could be determined by employing a unit root test. The power of such a test is, however, not undisputed and the reason to use such a test is that usually the innovation $v_{i,t}$ cannot be identified. In our case, as in the Japanese case of Davis and Weinstein, the innovation can be identified as long as we have valid instruments for the war shock ($s_{i,1946} - s_{i,1939}$) that serves as $v_{i,t}$ in the estimations for our German case below. The basic equation to be estimated for the case of a unique equilibrium then becomes:

$$s_{i,1946+t} - s_{i,1946} = \alpha(s_{i,1946} - s_{i,1939}) + \beta_0 + Z_i + error_i^8 \quad (4)$$

⁸ Note that we can include a constant because the summation over all s is not equal to 1 (the share of a city is relative to the total population, and not to the sum of city sizes in our sample).

where $\alpha = \rho - 1$, β_0 is a constant and Z_i refers to control variables one might want to include.⁹ If $\alpha = 0$ city growth follows a random walk. If we find that $-1 \leq \alpha < 0$ this is evidence that a random walk must be rejected and hence that the war shock had no effect at all ($\alpha = -1$), or at most a tendency towards a mean reverting process ($-1 < \alpha < 0$) on relative city growth in Germany. In our two-period growth setting of Fig. 1, $-1 \leq \alpha < 0$ implies that the shock has had a permanent effect, i.e. that the impact of the shock has not fully dissipated over time.

To estimate Eq. (4) we have to choose a t for the variable $s_{i,1946+t} - s_{i,1946}$. We choose $t = 17$ (indicating 1963) for our sample of West German cities which corresponds with the time horizon used by Davis and Weinstein [5]. Our implicit assumption here is that the period 1946–1963 should be long enough for the war shock to be fully absorbed. Also, the early 1960s mark the end of the reconstruction policies of the (West) German government.

Following Fig. 1, the question is now how to change the empirical specification to allow for the possibility to test for multiple equilibria. To simplify matters, assume for the moment that we know the number of equilibria as well as the thresholds, like b_1 and b_2 in Fig. 1. After we have introduced the model using these two assumptions, we will need to relax them in our empirical specification of the model with multiple equilibria. The problem is how to adapt Eq. (3) in order to take multiple equilibria into account. As a possible solution, assume for the moment (and in accordance with Fig. 1) that there are three stable equilibria. The initial equilibrium is denoted (in log-share) by Ω_i , the second or low equilibrium is $-\Delta_1$ (log-)share units below the initial equilibrium (this equilibrium is located at $\Omega_i + \Delta_1$ in log-share space; recall Fig. 1 from the previous section), and a third or high equilibrium at $\Omega_i + \Delta_3$. The lower and higher thresholds are denoted by b_1 and b_2 (in log-shares) respectively. For $b_2 > b_1$ we get for $s_{i,t+1}$:

$$s_{i,t+1} = \Omega_i + \Delta_1 + \varepsilon_{i,t+1}^1 \quad \text{in case } v_{i,t} < b_1, \quad (5a)$$

$$s_{i,t+1} = \Omega_i + \varepsilon_{i,t+1}^2 \quad \text{in case } b_1 < v_{i,t} < b_2, \quad (5b)$$

$$s_{i,t+1} = \Omega_i + \Delta_3 + \varepsilon_{i,t+1}^3 \quad \text{in case } v_{i,t} > b_2. \quad (5c)$$

By subtracting (1) from (5), and also using the fact that the error terms change whenever the lower or upper threshold b_1 or b_2 is crossed, it can be shown (Davis and Weinstein [6, pp. 21–23]) that Eq. (3) becomes

$$s_{i,t+1} - s_{i,t} = \Delta_1(1 - \rho) + (\rho - 1)v_{i,t} + [v_{i,t+1} + \rho(1 - \rho)\varepsilon_{i,t-1}] \quad \text{in case } v_{i,t} < b_1, \quad (6a)$$

$$s_{i,t+1} - s_{i,t} = (\rho - 1)v_{i,t} + [v_{i,t+1} + \rho(1 - \rho)\varepsilon_{i,t-1}] \quad \text{in case } b_1 < v_{i,t} < b_2, \quad (6b)$$

$$s_{i,t+1} - s_{i,t} = \Delta_3(1 - \rho) + (\rho - 1)v_{i,t} + [v_{i,t+1} + \rho(1 - \rho)\varepsilon_{i,t-1}] \quad \text{in case } v_{i,t} > b_2. \quad (6c)$$

Equation (6) replaces Eq. (3) in our example with three equilibria. Note that Eq. (6b) is exactly the same as Eq. (3). Note also that Eqs. (6a)–(6c) are the same except for their constant term. As a final step, and taking into account that for our case of German cities $t = 1946$, $t + 1 = 1963$ and

⁹ Note that the measure of the shock (or innovation) is the growth rate between 1939 and 1946, which is correlated with the error term in the estimating equation. This indicates that we have to use instruments.

that the war shock $v_{i,t} = v_{i,1946}$ will be approximated (see Eq. (4)) by the wartime city growth ($s_{i,1946} - s_{i,1939}$), we follow Davis and Weinstein and rewrite Eq. (6):

$$s_{i,1963} - s_{i,1946} = (1 - \rho)\Delta_1 I_1(b_1, v_{i,1946}) + (1 - \rho)\Delta_3 I_3(b_2, v_{i,1946}) + (\rho - 1)(s_{i,1946} - s_{i,1939}) + [v_{i,1963} + \rho(1 - \rho)\varepsilon_{i,1933}] \quad (7)$$

where $I_1(b_1, v_{1946})$ and $I_2(b_2, v_{1946})$ are indicator variables that equal 1 if $v_{i,t} < b_1$ or $v_{i,t} > b_2$ respectively.

Equation (7) is the multiple equilibria version of Eq. (4). Assuming that we have valid instruments for wartime city growth, we are now in a position to estimate Eq. (4) and hence to test the unique equilibrium hypothesis. However, this is not the case for Eq. (7) and the multiple equilibria hypothesis. The main problem with the latter is that we have assumed that the number of equilibria is known (three in our example) and also that we know the value of the thresholds b_1 and b_2 . But this has to be determined by the data, and not assumed beforehand.

Estimating a log likelihood function that is maximized for the values of the thresholds, and which yields coefficients for Δ_1, Δ_3 seems to be the appropriate methodology to estimate (7). Davis and Weinstein [6, p. 23] argue that this is problematic given that one also needs to instrument wartime growth, and the instrumenting equation would then include the indicator variables, which are in turn a function of the instrumented shocks. To get around this problem, they impose $\rho = 0$ which means, in our case of German city growth and the WWII shock, that whatever happens during the period of the shock (1939–1946) is undone in the second period (here 1946–1963). In our view, the need for the $\rho = 0$ assumption already follows directly from the underlying theoretical model as summarized by Fig. 1. Under the assumption that the period $t + 1$ in Fig. 1 is long enough for the shock from period t to have dissipated, a solid line in Fig. 1 represents a stable equilibrium with slope minus unity because that is what a *stable* equilibrium implies: $\Delta S_t = -\Delta S_{t+1}$. Any other assumption than $\rho = 0$ is difficult to ground on the underlying model of city growth. Thus, the (very important) assumption that $\rho = 0$ is already a consequence of the (implicit) theoretical model as summarized by Fig. 1. With $\rho = 0$, Eq. (7) becomes:

$$s_{i,1963} - s_{i,1939} = \Delta_1 I_1(b_1, v_{i,1946}) + \Delta_3 I_3(b_2, v_{i,1946}) + [v_{i,1963} + \rho(1 - \rho)\varepsilon_{i,1933}]. \quad (8)$$

Compared to Eq. (7), the relative city growth during the WWII period, $s_{i,1946} - s_{i,1939}$, has dropped out and one can now estimate Eq. (8) by selecting values for the parameters that maximize the likelihood function.

5. Data set and estimation results

5.1. Data set

Our data set is the same as in Brakman, Garretsen and Schramm [4]. We therefore refer the reader to that paper for more details on the bombing and destruction of German cities during WWII. The sample consists of cities in the territory of present-day Germany that either had a population of more than 50,000 people in 1939 or that were in any point of time in the post-WWII period a so called *Großstadt*, a city with more than 100,000 inhabitants. This yields a sample of 103 cities in total, consisting of 81 West German and 22 East German cities. Based on our previous work we decided to restrict the analysis to the sub-sample of West German cities. There are a number of reasons for doing this. First, the sub-sample for East German cities is rather small and data on war destruction are not as readily available for the GDR period. Secondly, and

more importantly, in the post-WWII period East German cities were part of the communist GDR with its central planning system in which economic agents were not free to choose their location. This is very much at odds with the basic theory, see Section 2, which underlies our empirical specification.

To analyze post-WWII city growth, we need cross section data on the WWII shock and time series data on city population. With respect to the former, Kästner [13] provides West German cross section data about the loss of housing stock in 1945 relative to the housing stock in 1939, and rubble in m^3 per capita in 1945. Data on the number of war casualties are only available for a sub-set of our cities and are, as opposed to the case of Japan, probably not a good indicator of the war shock.¹⁰ This leaves us with two variables that measure the degree of destruction.

Time series data of city population are from the various issues of the *Statistical Yearbook*, and for 1946 also from the *Volks- und Berufszählung vom 29 Oktober 1946 in den vier Besatzungszonen und Groß-Berlin*. As we will run regressions on the relative size of cities before and after WWII (city size relative to the total population), we also need statistics on the national population. This is not as easy as it might seem because the German border did change after WWII. There are *pre*-WWII time series of population for the part of Germany that became the Federal Republic of Germany (FRG), that is West Germany, in 1949 (*Federal Statistical Office*). To allow for the impact of geography we included the geodesic distance between cities (see below). Finally, as an alternative to city population we use data on so called *Gewerbesteuer* (corporate taxes) on the city level to measure city growth. These tax data were also taken from *Statistical Yearbook* and are an indicator of the degree of economic activity.

5.2. The case of a unique equilibrium and the relevance of geography

As we explained in the previous section, for the case of a unique equilibrium we estimate Eq. (4) where we have to instrument relative city growth during WWII: $s_{i,1939} - s_{i,1946}$. The city-specific instruments are the destruction of the housing stock between 1939 and 1945, and m^3 rubble per capita in 1945. In addition, we introduce the role of geography in two ways:

- (1) The relative city growth from 1939–1946 is not only instrumented by its own destruction but also by a sum of the distance-weighted housing destruction and rubble variables of all other cities. In a very simple way this captures the idea that the “geography” of bombing may matter, in the sense that the city growth of city i during WWII is also determined by the (exogenous) degree of destruction of other cities corrected for distance, see also Fig. 1. Hence, as opposed to Davis and Weinstein [6], the relative city growth of a city is no longer only a function of city-specific variables, that is to say it is no longer independent of the

¹⁰ For the German case it is in our view not straightforward to include the number of casualties per city to measure the degree of destruction. Systematic data on casualties are lacking and if they were available they include prisoners of war (PoWs), foreign workers (*Fremdarbeiter*), and refugees and are therefore not a good indicator of the destruction of a city. For a sub-set of cities, based on Friedrich [8], we have city data on casualties and from these data we know that PoWs, refugees and foreigners often contributed more than proportionally to a city’s death toll (they were often denied shelter during bombardments). The distribution of these “temporary” inhabitants of a city was often not linked to the size of the city. This means that, as opposed to the case of Japan as analyzed by Davis and Weinstein [5,6], the number of casualties is probably not as good an indicator of city destruction as the change in the housing stock or rubble in m^3 per capita.

fate of other cities.¹¹ Linking cities in this manner is not a problem since the war shock is assumed to be exogenous. We label this extension GEO-I.

- (2) In Eq. (4) we add the distance between city i to one of the three West German economic centers (Hamburg, Köln, München) as a control variable, so as to capture the idea that for post-war city growth $s_{i,1963} - s_{i,1946}$ geography could matter as well. In line with, for instance, a simple market potential function or more elaborate new economic geography models, the distance from economic centers is thought to be able to affect a city's growth rate. We label this extension GEO-II.

Without geography, the estimation of (4) using our two instruments should yield the same results as in Brakman, Garretsen, and Schramm [4] for the case of West German cities. This is indeed the case.¹² Table 1 presents the estimation results for the model specification with a unique equilibrium. The upper panel gives the 1st stage estimation results and the lower panel the 2nd stage results. The various columns in Table 1 differ in their use of the geography extensions GEO-I and GEO-II.

In addition to the variables introduced above, a dummy for Berlin has been added in specifications where geography matters. This amounts to excluding Berlin from the data set, as (the western part of) this city in the period under consideration was isolated from the rest of West Germany. München is included as our economic center. The estimations suggest that cities closer to München grow faster.¹³ We also looked for a possible effect of the former FRG–GDR border in the sense that we included the distance between a West German (FRG) city and the border of East Germany (the GDR). The results were, however, insignificant once the Berlin dummy was added to our set of explanatory variables in Eq. (4).¹⁴

As for the relevance of geography in the 1st stage regression (the war shock), see Table 1a, we find that the distance decay parameter is significant and around -2 . The latter implies that the impact of the war shock of another city on the city growth of city i quickly becomes quite small when the distance between city i and the other city, D_{ij} , increases. For city i , it is above all the

¹¹ Without geography WWII city growth is instrumented in a 1st stage regression as follows:

$$s_{i,1946} - s_{i,1939} = \mu + \phi_1 \ln(\text{housing stock})_i + \phi_2 \ln(\text{rubble})_i + \psi Z_i + \varepsilon_i,$$

where Z are the exogenous variables, if any, in Eq. (4). With geography we get:

$$s_{i,1946} - s_{i,1939} = \mu + \phi_1 \ln\left(\sum D_{ij}^\delta \text{housing stock}\right)_i + \phi_2 \ln\left(\sum D_{ij}^\delta \text{rubble}\right)_i + \psi Z_i + \varepsilon_i,$$

where D_{ij} is the distance between city i and city j , and δ is the distance decay parameter that needs to be estimated. Davis and Weinstein [6, p. 9] acknowledge that the fact that the relative growth of a city is independent of the evolution of other city sizes as a limitation of their approach.

¹² With one caveat: for the actual estimations growth rates are constructed using $(x_t - x_{t-i})/x_{t-i}$ instead of the approximation $\ln(x_t) - \ln(x_{t-i})$ that we, following Davis and Weinstein [5], used in our previous paper. In their discussion of these papers, Head and Mayer [12] rightly stress that this approximation is only valid for small changes and large changes cannot be ruled out.

¹³ When we took Köln as the economic center, the resulting coefficient was not significant, and when we opted for Hamburg, the distance coefficient became $+0.0003^{***}$ which supports the idea that cities in the South grew faster.

¹⁴ Redding and Sturm [15] find, however, evidence that supports the relevance of the FRG–GDR split for border regions in West Germany (FRG). Their approach differs in a number of important ways from our analysis so it is difficult to compare these findings. Redding and Sturm [15] use a different sample of cities, include smaller cities and instead of, as we do, using distance to the GDR border as an explanatory variable they divide their sample into two groups of cities: West German cities that are near (within 75 km range) to the GDR border and cities that are not near to this border, and then test for differences in the growth performance between these two groups of cities.

Table 1a
1st stage regression results

$s_{i,1946} - s_{i,1939}$	No geography	GEO-I	GEO-II	GEO-I + GEO-II
Constant	0.203***	0.210***	0.138**	0.164**
Distance decay, δ	–	–2.215***	–	–1.918***
ln(housing)	–0.055**	–0.051*	–0.079***	–0.078***
ln(rubble/capita)	–0.096***	–0.104***	–0.086***	–0.098***
DummyBerlin	–	–	–0.112	–0.119
DistanceMünchen	–	–	0.0003***	0.0003***
Adj. R^2	0.629	0.629	0.693	0.699

Table 1b
2nd stage regression results

$s_{i,1963} - s_{i,1946}$	No geography	GEO-I	GEO-II	GEO-I + GEO-II
Constant	0.099***	0.101***	0.210***	0.212***
$s_{i,1946} - s_{i,1939}$ ($\alpha = \rho - 1$)	–0.550***	–0.538***	–0.627***	–0.617***
DummyBerlin	–	–	–0.393**	–0.392**
DistanceMünchen	–	–	–0.0003***	–0.0003***
SER	0.167	0.167	0.153	0.153

Note. “–” indicates that the variable is not relevant for that model variant.

* Significance at the 10% level.

** Idem, 5% level.

*** Idem, 1% level.

own destruction that matters and both instruments (housing and rubble) are highly significant. All in all the conclusion is that the geography variables are significant but that they do not change the main result as to the coefficient for the instrumented wartime city growth $s_{i,1946} - s_{i,1939}$. This coefficient, the α -coefficient in Eq. (4), is our main interest here (see Table 1b). Recall from Section 4 that $\alpha = \rho - 1$ and we thus find that the estimated coefficient is clearly significantly different from -1 which would have been an indication of complete mean reversion in 1963. We find mean reversion in the sense that the average West German city recaptured about 50% of the war shock (positive or negative in relative terms) by 1963. So there is a tendency to return to the pre-war growth path but this tendency, as opposed to the findings of Davis and Weinstein for Japan, is far from complete. Adding government reconstruction expenses or pre-war city growth as city-specific control variables did not change our results.

Table 1b shows that adding geography, and hence allowing for spatial interdependencies between cities, does not change the conclusions from Brakman, Garretsen, Schramm [4] despite the fact that the geography variables are significant. The question is of course whether this conclusion also holds when we add geography to the multiple equilibria framework. Before we address this question, a final remark on how our findings in Table 1 relate to the corresponding estimation results by Davis and Weinstein. They find for Japan that the α -coefficient is not statistically different from -1 which thus means that the presence of a unique equilibrium is borne out by the Japanese data. It is the case for Japan that the observations in the S_t, S_{t+1} space of Fig. 1 can be arranged on a line through the origin with slope minus unity at the (initial) equilibrium Ω . The latter is thus not the case for our sample of West German cities. Given this difference one may argue that it is more interesting for the German case to start to look for multiple equilibria than it is for the case of Japan. Davis and Weinstein [6, p. 10] note that a finding (like in our

Table 1) that α is significantly different from -1 is sufficient to establish that multiple equilibria are possible.

5.3. Multiple equilibria and the relevance of geography

In Section 4 we arrived at Eq. (8) as the specification to test for multiple equilibria. In line with the model underlying Fig. 1, we assume that $\rho = 0$ (or in terms of Table 1, $\alpha = -1$). This assumption means that the war shock (the 1st period, here 1939–1946) has completely been reversed in the subsequent period (here 1946–1963). Before we can present our estimation results for the multiple equilibria *cum* geography case, we will first briefly explain the estimation strategy employed by Davis and Weinstein to estimate their equivalent of Eq. (8) for Japan. Using the model underlying Fig. 1, and taking the case of three equilibria as our example,¹⁵ we now rewrite Eq. (8) as follows:

$$s_{i,1963} - s_{i,1939} = \delta_2 + \delta_1 I_1(b_1, v_{i,1946}) + \delta_3 I_3(b_2, v_{i,1946}) + \gamma Z_i + \mu_i. \quad (9)$$

Compared to Eq. (8) a constant term has been added to allow for a non-zero mean error term. Z_i stands for control variables like the government reconstruction expenses or the pre-war growth rate for city i that one might want to include, δ_1 , δ_2 , and δ_3 are parameters to be estimated and μ is the error term. The constant, δ_2 , represents the equilibrium in Fig. 1 for the line that cuts through the origin with slope minus unity, denoted by Ω . The fact that δ_2 (the constant term) represent the second equilibrium from the left in terms of Fig. 1 is nothing but a normalization. In line with the theoretical design, larger negative (positive) shocks should push a city to smaller (higher) equilibria, denoted by Δ_1 and Δ_3 in Fig. 1 respectively. This is captured in the estimation of (9) with b_1 (b_2) representing the minimum size of a negative (positive) shock that is needed to push a city to a smaller (larger) city size equilibrium ($\delta_1 + \delta_2$ and $\delta_3 + \delta_2$ respectively). This set-up, and Fig. 1, shows that the ordering of the intercepts should be such that (assuming there are at most three equilibria)¹⁶:

$$\delta_1 + \delta_2 < \delta_2 < \delta_2 + \delta_3 \quad \Leftrightarrow \quad \delta_1 < 0 < \delta_3.$$

A second requirement from the theory as summarized by Fig. 1 is that the thresholds b_1 and b_2 lie between the equilibria. This means that we get the following final intercept and threshold ordering condition:

$$\delta_1 < b_1 < 0 < b_2 < \delta_3. \quad (10)$$

Keeping Eq. (10) in mind, Eq. (9) is estimated by maximum likelihood through the use of a grid search technique as proposed by Davis and Weinstein [6, pp. 25–26]. This technique is very similar in spirit to performing a threshold regression with unknown thresholds (see Hansen [10]). The Davis and Weinstein estimation strategy involves several steps. First, as in the case of a unique equilibrium, the relative population growth of each city between 1939 and 1946 is regressed on our preferred war-related destruction variables (housing stock destruction in 1945 and m^3 rubble per capita in 1945) with and without our geography extension GEO-I. Given the estimated coefficients from this regression, we then construct the “fitted warshocks” for each city by multiplying

¹⁵ As with Fig. 1, the case of three equilibria is merely chosen for illustrational purposes. The estimation procedure is the same when considering two or more than three equilibria.

¹⁶ The first equilibrium intercept is Constant + δ_1 ; the second equilibrium: Constant (= δ_2); the third equilibrium: $\delta_2 + \delta_3$; etc.

the housing stock destruction and rubble variables, possibly using the GEO-I extension, by their respective estimated coefficients. Next, we group the cities in our sample into three groups on the basis of the magnitude of their “fitted warshock” and two threshold values.¹⁷ As these threshold values are, however, themselves parameters to be estimated, they could in theory take on any value. Following Davis and Weinstein [6], we use the following procedure to pinpoint these two parameters: calculate every value of the “fitted warshock” (constructed in the 1st step) that corresponds to a percentile of its empirical distribution that is a multiple of 5, i.e. corresponding to the 5th, 10th, . . . , 95th percentile (other multiples are also possible but that would lead to level of detail not supported by the data). Then group the cities in the sample in three groups using as threshold values, b_1 and b_2 , any pair of “fitted warshock” values that corresponds to two 5% percentiles of the war shock and then estimate (9). That is, estimate $\sum_{n=1}^{20} (n - 1)$ different specifications with the thresholds (b_1, b_2) being the values of the “fitted war shock” corresponding to the (5th, 10th), (5th, 15th), . . . , (5th; 95th) percentiles, then do the same for the (10th, 15th), (10th, 20th), . . . , (10th, 95th) percentiles, etc., until the (90th, 95th) percentile of the empirical distribution of the “fitted warshock” variable. For each of these estimations, store the value of the likelihood function and the estimated parameters, δ_1 , δ_2 , and δ_3 . Those threshold values and parameter values that correspond to the maximum value of the likelihood function give the estimation result that, for the number of equilibria under consideration (here 3), is shown in Table 2.

We incorporate the role of geography in Table 2 above in a similar way as for the case of a unique equilibrium reported in Table 1. In constructing the war shock data that is the basis for the grid search, we ran regressions in which only the own city destruction entered (the No Geo columns in Table 2) and regressions in which we took geography into account by letting the destruction of other cities (weighted by distance) also be a determinant of the city growth in the period 1939–1946 (the Geo columns in Table 2). Finally, given the significance of the geography variables in the 2nd stage regression as shown in Table 1b we included the Berlin dummy and the distance to München in our set of control variables when estimating Eq. (9).

Table 2 does not show the results for the case of a single equilibrium since we already know from Table 1 what the conclusions are.¹⁸ For the case of two equilibria (1st and 3rd column) we find that $\delta_1 < 0$ which is what theory tells us to expect. Larger negative shocks push cities to smaller equilibria. But the ordering criterion, which also takes the threshold restrictions into account ($\delta_1 < b_1 < 0$, see last row of the table), *is only met when we allow geography to play a role*. So there is evidence in favour of two equilibria. Furthermore the LR-test, see bottom of Table 2, indicates that the null hypothesis of a unique equilibrium versus the alternative of two equilibria is always rejected (p -value: 0.00). Given the fact that the estimations in Table 2 are based on the assumption of $\rho = 0$ ($\alpha = -1$) evidence in favour of multiple equilibria is less of a surprise if one takes into account that we found α to be around -0.6 in Table 1b for the case of a unique equilibrium. This also helps to explain why Davis and Weinstein find no evidence in favour of multiple equilibria (given that they already established that α is (almost) -1 when estimating Eq. (4) for city population in Japan).

¹⁷ In general, group the cities in n groups on the basis of $n - 1$ thresholds when considering n equilibria.

¹⁸ When we did estimate Eq. (9) for the case of one equilibrium we found, not very surprisingly, that coefficients for the three variables (constant, Berlin dummy, distance to München) are somewhat different to those reported in Table 1, the reason being that we now impose that $\rho = 0$ ($\alpha = -1$) and the estimation results shown in Table 1 indicate that ρ is not zero ($\alpha = -0.6$) for Germany. In our estimations without the Berlin dummy and the distance to München, the evidence in favour of the “GEO-II equilibrium” model is less clear-cut.

Table 2
 Estimation results for multiple equilibria and the role of geography

$s_{i,1963} - s_{i,1939}$	No Geo, 2 equilibria	No Geo, 3 equilibria	Geo, 2 equilibria	Geo, 3 equilibria
Constant (= δ_2)	0.219***	-0.055	0.284***	0.458***
Dummy _{Berlin}	-0.389***	-0.419***	-0.419***	-0.413***
Distance _{München}	-0.0003***	-0.0002**	-0.0002**	-0.0003***
δ_1	-0.141***	0.134**	-0.193***	-0.341***
b_1 threshold	-0.191	-0.234	-0.004	-0.004
δ_3	-	0.249***	-	-0.220***
b_2 threshold	-	-0.191	-	0.017
Log likelihood	38.389	41.074	38.766	41.710
AIC, Akaike	-0.893	-0.937	-0.903	-0.954
SBC, Schwartz	-0.771	-0.785	-0.781	-0.801
Intercept order criteria (Eq. (10)), 2 equilibria: $\delta_1 < b_1 < 0$ 3 equilibria: $\delta_1 < b_1 < 0 < b_2 < \delta_3$	Fail	Fail	Accept	Fail

LR-stat (# equilibria)	No Geo	Geo
1 vs. 2	15.144	15.898
2 vs. 3	5.370**	5.888**

Notes. (1) Test statistic is distributed as $\chi(1)$.

(2) The likelihood ratio (LR) test rejects one equilibrium against two equilibria for all specifications. When two equilibria are tested against three equilibria, this is accepted at the 5% level.¹⁹

* Significance at the 10%.

** Idem, 5%.

*** Idem, 1%.

When one allows for three equilibria it is true both in the geography and the non-geography case (4th and 2nd column respectively) that the δ -coefficients are significant but that the restriction, $\delta_1 < 0 < \delta_3$, only holds for the non-geography case. For this case the threshold criterion is not met (last row of Table 2). In addition, the LR-test indicates that we cannot reject the null of two equilibria versus the alternative of three equilibria at the 1% level (p -value: 0.02).²⁰ So there is no firm evidence to support the model with three equilibria.

Our overall conclusions with respect to Table 2 are twofold. First, there is evidence in favour of multiple equilibria for the case of German city growth. If we take the ordering restrictions that follow from Fig. 1 into consideration, we conclude that the German case is best described by two equilibria and also that this two equilibria specification is to be preferred to the case of a unique equilibrium.²¹ This is in contrast with the case of Japan as analysed by Davis and Weinstein.

¹⁹ A different approach to the same problem (testing for a threshold) can be found in Hansen [10,11]. Using Hansen [10] we can test for the presence of one threshold in the war shock data for our sample of West German cities. This requires that all parameters are allowed to be different for the two sub-groups, this is only possible for our specification when we do not include the Berlin dummy or the distance to München variable. We then find evidence in favour two equilibria (instead of one equilibrium).

²⁰ Results for more than three equilibria did not improve matters and never met the ordering criteria.

²¹ The maximum likelihood for the two equilibria case with geography was achieved for an 85–15% split of the war shock data. This means, with a threshold of $b_1 = -0.004$ (see Table 2), that the following twelve relatively less “war struck” cities remained in the initial equilibrium Ω of Fig. 1: Flensburg, Neumünster, Lübeck, Herne, Recklinghausen, Leverkusen, Bergisch-Gladbach, Göttingen, Heidelberg, Erlangen, Regensburg, and Ingolstadt.

Second, the preference for the two equilibria specification crucially depends on the inclusion of geography in the construction of the city-specific war shock variable. Without geography, the evidence for multiple equilibria is much weaker because the ordering criteria are not met. So, geography really matters. This shows that it makes a difference whether or not spatial interdependencies between cities are allowed to play a role. Our operationalisation of geography is very simple, and can no doubt be improved upon, but our results suggest that its incorporation matters.

6. Further research and conclusions

6.1. Further research

A more elaborate analysis of geography within the multiple equilibrium framework of this paper would be an interesting topic. Apart from this, there at least three other options for further research. From Section 2 we know that the new economic geography model by Krugman [14] is the (implicit) benchmark for the estimations. The ordering criteria that follow from this model and hence from Fig. 1 are clearly very important for our conclusions with respect to the relevance of multiple equilibria (this also holds for the conclusions in Davis and Weinstein [6] about Japan).²² Without these criteria, conclusions as to the preferred model are different (see the Akaike (AIC) and Schwartz (SBC) information criteria in Table 2). More information is needed to tell us whether and how restrictions like those given by Eq. (10) are consistent with other NEG models.

A second line for additional analyses deals with the choice of the dependent variable. One of the innovations of Davis and Weinstein [6] is that they not only test Eqs. (4) and (9) with city population as the city growth variable but they are (quite uniquely) able to do similar estimations with total and sector data on manufacturing activity as indicators of city growth. This is interesting because the location behaviour of citizens and firms following a shock like the WWII destruction of cities might be quite different. Unfortunately, city specific data on economic activity for the period under consideration are not available for German cities. The best we could come up with was the use of city data on *Gewerbesteuer* (corporate taxes). Correcting for differences in tax rates between cities and over time, we took changes in a city's corporate tax income (relative to the corresponding corporate tax income for West Germany as a whole), as an indicator of the relative change of the degree of economic activity for each city in our sample. Using the relative change of cities' corporate tax receipts, we estimated the same specifications as to those underlying Tables 1 and 2 for relative population growth. The results are in line with those reported in Tables 1 and 2 (results not shown here but see Bosker et al. [1, pp. 26–27]). When estimating Eq. (9) with the relative change in cities' *Gewerbesteuer* we even find evidence for the case of three equilibria, notably when geography is added.²³

As a third suggestion for future research, Germany in the 20th century provides other potentially interesting shocks besides WWII. For instance, the split of Germany in the FRG and the GDR after WWI and the subsequent re-unification of Germany in 1989 come to mind, as well as the impact of WWI and the turmoil between 1918 and 1939. In order to analyse these shocks,

²² See for instance Davis and Weinstein [6, p. 33] where without the ordering criteria one would pick the model with two equilibria as the preferred model.

²³ One way to find more economic indicators at the German city level (see Redding and Sturm [15]) might be to try to make use of Census-data that provide city-specific information on the production structure and employment.

the present framework will, however, not do because of the fact that these shocks are not city-specific (like the FRG–GDR split in 1949). Different techniques like (panel) unit root tests are called for to analyse the multitude of shocks that hit German cities in the past century. In Bosker et al. [2] we apply panel unit root tests on annual German city population data that cover (almost) the whole 20th century. Here too we find a clear (and permanent) impact of the WWII episode.

6.2. Conclusions

Many modern location models are characterized by multiple equilibria, but it is difficult to test for the presence of multiple equilibria. Davis and Weinstein [5] and Brakman, Garretsen and Schramm [4] study the effects of a large exogenous and temporary shock (WWII) on city growth for the case of Japan and Germany respectively. They both find evidence that the WWII shock tended to undo itself over time, although only partially for the case of Germany. This would suggest that city growth is not characterized by multiple equilibria. Based on the methodology developed by Davis and Weinstein [6] for the case of Japanese cities and WWII, we test explicitly for multiple equilibria for West German cities in the present paper. In doing so, and in addition to the Davis and Weinstein framework, we look at the spatial interdependencies between cities. The main findings are twofold. First, multiple equilibria seem to be present in the evolution of the German city size distribution. We find evidence of a model with two stable equilibria. Second, the explicit inclusion of geography matters. Evidence for multiple equilibria is weaker when spatial interdependencies are not taken into account.

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Appendix A. New economic geography (NEG) and multiple equilibria

Figure A.1 depicts the equilibrium allocation of the footloose firms and workers in a two-region NEG model where the Krugman [14] model is changed by adding congestion as an additional spreading force (see Brakman et al. [3, Chapter 7]). In such a model, one may end up with multiple stable interior equilibria. The example depicted in Fig. A.1 displays five equilibria where a (long-run) equilibrium occurs when the ratio of real wages for regions 1 and 2, w_1/w_2 , equals one. With real wage equality, footloose workers have no incentive to migrate. Figure A.1 gives five equilibria (a – e). Equilibria a , c and e are stable equilibria: if a worker would migrate, the resulting real wage change is such that she has an incentive to return. Equilibria b and d are unstable long-run equilibria: a relocation of workers leads to a real wage change that validates the migration decision resulting in more workers to migrate. If initially the economy finds itself between two equilibria, like between equilibria b and c where $w_1/w_2 > 1$, migration of workers from region 2 to region 1 will ensure that we end up in the stable equilibrium c . For our present purposes and to illustrate the connection between Fig. A.1 and Fig. 1 in the main text, note that for an equilibrium to be stable we need a negative slope around this equilibrium (here a , c and e) and that unstable equilibria (here b and d) can be looked upon as thresholds where one passes from one stable equilibrium to another. Assuming that the economy is initially in a stable equilibrium (like c), a shock to the footloose production factors implies the following. Either the

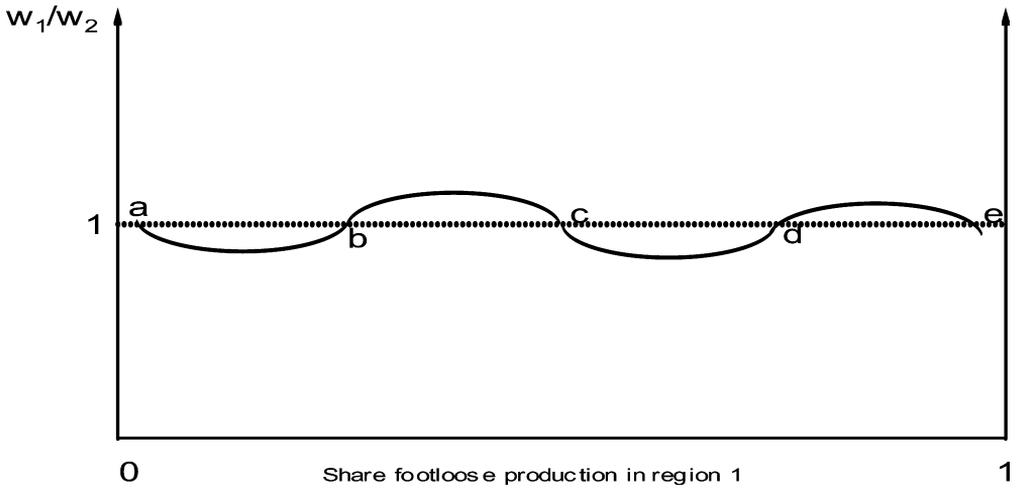


Fig. A.1. Multiple equilibria in an NEG setting.

shock is small and footloose workers and firms return to their initial location (c) or the shock and the resulting relocation of workers and firms is large enough to pass the threshold of the nearest unstable equilibrium (here b or d) and the economy moves to another stable equilibrium (here a or e). The mapping between Fig. A.1 and Fig. 1 is such that the stable equilibria a , c and e and the unstable equilibria b and d in Fig. A.1 correspond with the stable equilibria Δ_1 , Ω , Δ_2 and the thresholds b_1 and b_2 in Fig. 1 respectively. As we explained in Section 2, for an equilibrium to be stable it must be the case in the two-period growth set-up of Fig. 1 that there exists a period $t + 1$ that is long enough such that $\Delta S_t = -\Delta S_{t+1}$. To see this, look at Fig. A.1: if, starting from the initial, stable equilibrium c , a shock occurs in period t that moves the economy to a point between b and c then ultimately the economy will have to return to equilibrium c which means that the effect of a shock at period t is *exactly off-set* in period $t + 1$.

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